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Economic Analysis

Simulations of long-term returns and replacement rates in the Colombian pension system

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Overview

This study is a theoretical exercise for Colombia that aims to simulate a variety of scenarios under a hypothetical scheme similar to the multi-funds currently in operation in Chile, Mexico and Peru. This has been done by modeling the future movement of asset prices that are considered to be representative of equity and fixed-income using the Monte Carlo method. After making the simulations we have constructed alternative investment portfolios according to the chosen combination of equity and fixed-income, and compared and assessed them in terms of their risk-return ratio. The study emphasizes the fundamental importance of adequate contribution densities for obtaining sufficient income for old age, and the relevance of high returns, with adequate risk limitation. Another of the study's aims is to use the new multi-fund scheme defined for Colombia as a basis for the hypotheses of different scenarios projected to 2050. These will include the composition of members' pension fund portfolios and changes in the scheme over time, taking as a reference the life-cycle scheme operated in Mexico, as well as other compositions and profiles that participants may decide to enter into in accordance with their choice and the limits set by regulations. The results of the work confirm what has been found in other studies on the subject for the Colombian economy: that the implementation of a multi-fund system will provide pension-fund members with efficient returns in the long term, with limited volatility over time.

1. Introduction

Since its creation in 1993, the Colombian pension system has been composed of two sub-systems: a pay-as-you-go system, called Average Premium Scheme (RPM) and an individual capitalization scheme called the Individual Saving with Solidarity Scheme (RAIS). The individual capitalization subsystem has been designed in such a way that in the accumulation phase participants share a common investment scheme for their assets, which they save in individual accounts. Currently, these funds are administered by six Pension Fund Administrators.

Although the general pension system has undergone some significant reforms, its initial structure of two parallel subsystems has remained unaltered. However, there have been important reforms in recent years, mainly focused on parametric changes in the case of the RPM. In the case of the RAIS, the biggest change has been the recent creation of a multi-fund scheme. This multi-fund scheme allows members' assets to be invested in the way that is best adapted to their risk-return profile, to the extent that it admits a variety of different investment portfolios for members.

The multi-fund system introduced by Law 1328 of 2009 must have 3 funds in its accumulation phase: a conservative fund, a moderate fund and a higher-risk fund. Pension fund members may choose their fund and transfer freely between them. They may also freely choose their administrator. The government will have to decide on the rules for assigning participants to funds, taking into account their age and gender and the contributions made. It also still has to define the rules for assigning participants to the moderate or conservative fund if they have not made the choice for themselves within the allotted time[1].

In addition, the Law establishes the following: (i) each fund's investment should take into account the types and proportion of assets admissible according to risk. The national government shall establish their investment rules with the aim of procuring the best risk-adjusted return for participants; (ii) there must be a minimum return applicable to the funds; (iii) there must be a gradual move towards the multi-fund scheme. The above makes it clear that the implementation of the multi-fund system requires a definition of key aspects for the system and for the participants that therefore require careful study before the decision-making stage.

This document aims to carry out a theoretical exercise that analyzes one of the important aspects to be defined under the new set of investment rules for multi-funds: their proportion of equity and fixed-income. We have done this by modeling the future movement of asset prices that are considered representative of equity and fixed-income using the Monte Carlo method. After making the simulations, we constructed alternative investment portfolios according to the chosen combination of equity and fixed-income, and assessed their relative risk-return ratio.

It is important to point out that the results obtained in the different scenarios should be considered as part of a series of theoretical exercises based on specific assumptions and limited by the characteristics of the information available. Thus the patterns demonstrated by the projections should be taken as reference and not as precise paths that the Colombian pension system will take under a multi-fund scheme. It should also be taken into account that at the completion date of the study, some central points of the law still had to be clarified.

This document is divided into 5 parts. The first part is the introduction. The second section reviews some of the existing studies that analyze the investment portfolio of pension funds in Colombia. The third section describes the performance of funds in a short-term perspective, taking the recent crisis of 2008 as the period of study. The performance of the funds is then analyzed in the long term from a historical perspective based on future forecasts. Finally, we present our conclusions.

1: Currently the Operational Decree for multi-funds is under discussion. This will define aspects such as the form in which the transition from the old system to the new is carried out, the extent of choice given to the members, the rules for transfer between funds, the default fund, etc. The Decree will also include a timeline for the transition and implementation of the new system.

2. Theoretical framework: the investment portfolio in Colombia

Some authors such as Jara et al. (2005), Jara (2006) and Reveiz et al. (2007) suggest that the regulatory framework in which fund investments are made limit their performance in terms of returns. Jara et al. (2005) mention as an additional limiting factor the restrictions imposed by the size of the capital market in Colombia. According to Reveiz (2008) regulations reduce the levels of the Sharpe ratio[2] and the benefits that the funds may achieve from diversification. Among the limitations imposed by the regulation, the authors mention the calculation of the minimum return. Laserna (2007) proposes an alternative form of calculating the minimum return required for the funds, in order to reduce the restrictions that the current calculation imposes on investment decisions. Reveiz et al. (2008) also state that regulatory restrictions reduce expected returns and restrict both higher levels of return by unit of risk and the benefits of diversification. They propose that the multi-fund scheme should be introduced to ensure that the investments can be better matched to the members' risk profile. The authors state that the differentiation of the funds and the assignment of participants to them should answer the principle of maximizing wealth for a given level of risk, which minimizes the risk of not meeting expected liabilities (that according to the authors can be defined in terms of replacement rates). It should be noted that, in constructing the efficient portfolios, this study points to a strong concentration on domestic equity as the assets with greatest return, as the Colombian Stock Exchange Index (IGBC) had an unusually high return for the period of study (January 2001 - July 2007). Reveiz et al. (2009), using the returns of the efficient portfolios estimated by Reveiz et al. (2008b), estimate fund members' replacement rates and conclude, in terms of the performance of the pension system, that "equally important as the legal structure of the pension system and the creation of schemes such as multi-funds is the implementation of policies that promote adequate levels of coverage." The authors calculate expected returns using Wiener's generalized harmonic analysis methodology, and simulate the pensions obtained for different investment portfolios with different risk-return combinations.

Along similar lines to the above, this work demonstrates the fundamental importance of having adequate contribution densities in order to obtain sufficient income in old age. In fact, even with fairly high rates of return, participants with low contribution densities obtain reduced replacement rates. The exercises presented in this work show that the implementation of the multi-fund system will enable greater returns to be obtained by participants in the system, and that since these investments are long-term, they will present more limited returns. The research consists of estimating the long-term returns of portfolios with different combinations of equity and fixed-income, and based on these estimates, calculating the replacement rates obtained for different contribution densities. This will allow the different performances for various groups of participants to be observed, in accordance with the different assumptions used for the comparisons, as well as highlight the main trade-offs when the multi-fund scheme is implemented in the Colombian economy.

Among the main working assumptions that will allow us to calculate the different forecasting profiles is the case of implementing a similar scheme to that of the life-cycle multi-funds in Mexico. We will also simulate other profiles that give fund members a more flexible choice and try to replicate the choice profiles of other multi-fund schemes, such as those in Chile and Peru. This work has used the methodology followed by Herrera (2009) for the case of Mexico as the basis for developing this research.

2: The Sharpe ratio is the return per unit of risk.

3. Pension fund performance:

A) Short-term perspective: the impact of the financial crisis

In October 2008, pension funds had their lowest monthly yields of recent years, at -5.5.%. The loss between January and October 2008 was COP 1,47 trillion. These losses can be explained by the performance of the TES government bonds and the stock market, which are the main components of the fund portfolios. In fact, in 2008 the TES bonds accounted for nearly 40% of the portfolio and around 20% of domestic equity (see Table 1).

Table 1

Portfolio Composition: 2008

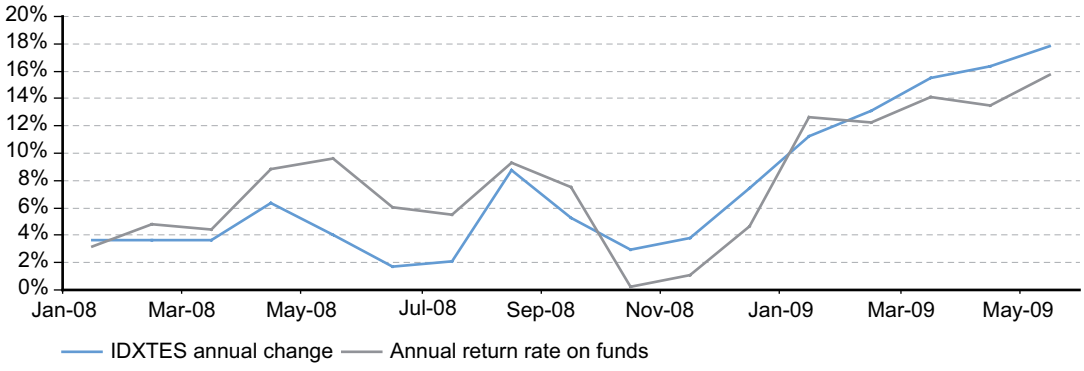
	jan-08	Apr-08	Jul-08	Oct-08	dec-08
Fixed income					
National	64.6	64.7	63.9	66.5	66.6
Treasury Bonds	40.9	41.7	40.7	40.6	41.39
Foreign	5.4	4.6	5.3	5.9	5.5
Total	**70**	**69.3**	**69.2**	**72.4**	**72.1**
Equity					
National	21.5	23.4	23.2	20.6	21.9
Foreign	6.2	5	4.6	4.4	3.9
Total	**27.7**	**28.4**	**27.8**	**27.8**	**25.8**

Source: Financial Superintendency.

In 2008, the IDXTES index of sovereign securities showed very small gains. In fact, in October 2008 this index had showed an increase of only 2.9% in the previous 12 months (see Chart 1).

Chart 1:

Annual change in the IDXTES and annual return of funds: January 2008 – June 2009



Source: Bank of the Republic and Colombian Stock Market, BBVA Research

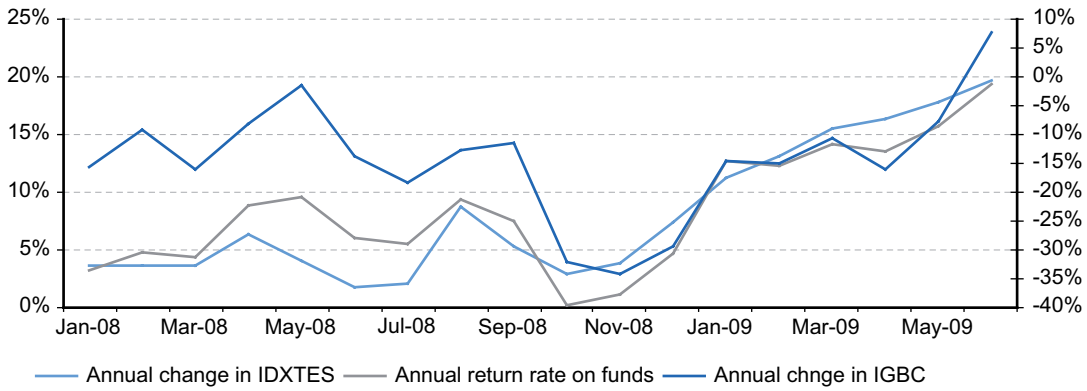
The above can be explained mainly by the increase in the medium- and long-term TES rates for the first 10 months of 2008, which meant a devaluation in their prices. Thus, for example, the 2020 bond increased from a rate of 10.3% in January to 13.2% at the close of October[3]. The medium- and long-term TES rates were influenced by greater inflationary expectations and by the increase in the levels of risk aversion internationally. In addition, in February and July the Board of the Bank of the Republic increased the interest rate, with the July rise being to 10%. Importantly, starting in October, both observed inflation and its expectations were reduced, leading to a reduction in TES interest rates. In addition, the Board of the Bank of the Republic reduced the interest rate.

3: Bank of the Republic of Colombia. Report of the Managing Board to the Congress of the Republic. March 2009.

As a result of this, the annual return of funds was at a low in October, and this was reinforced by the poor stock market performance. In fact, the IGBC fell by 29.3% from 10,694 in December 2007 to 7,560 in December 2008 (see Chart 2).

Chart 2:

Annual change in the IDXTES and IGBC indexes and annual fund rates of return



Source: Bank of the Republic, Colombian Stock Market, and Financial Superintendency of Colombia BBVA Research

The index's low was in November, when it touched 7,314.8. The biggest fall in the year was in October, at 24.7%, with an accumulated reduction in the previous 12 months of 34.2% in November.

Despite the major accumulated losses suffered by the funds over the first 10 months of 2008 (- COP 1.47 trillion), by December the balance was already positive, with an accumulated gain of COP 2.6 trillion (see Table 2). The historical return, in the worst moment of the crisis, stood at a nominal 13.4%, which shows that the losses were a short-term phenomenon. In 2009 the funds performed fairly satisfactorily, with an accumulated return net of fees of COP 13.9 trillion for January to September, and an accumulated return of 25.1%[4] . The accumulated returns of the funds from their start until September 2009 were COP 43.7 trillion. Between January 2008 and September 2009, the funds increased their accumulated returns by 54% in real terms (see Table 2).

4: This calculation uses NAV (net asset value) methodology to calculate the returns according to the change in the value of the fund. Corresponds to observations until September 2009.

Table 2

Initial balance, accumulated returns, returns for the period and final balance of pension funds in Colombia

(Trillions of pesos)	Initial balance	Net accumulated return on fees	Return on period	Final Balance
Jan-08	23.9	25.24	-1.942	49.6
Feb-08	24.3	25.51	0.270	50.3
Mar-08	24.8	25.59	0.082	50.8
Apr-08	25.2	27.78	2.189	53.4
May-08	25.6	28.22	0.441	54.3
Jun-08	26.0	26.97	-1.247	53.4
Jul-08	26.4	27.40	0.431	54.2
Aug-08	26.8	29.40	1.998	56.5
Sep-08	27.1	28.82	-0.579	56.3
Oct-08	27.4	25.71	-3.114	53.5
Nov-08	27.8	27.12	1.414	55.3
Dec-08	28.2	29.77	2.650	58.3
Jan-09	28.5	31.96	2.191	60.8
Feb-09	28.9	32.07	0.101	61.3
Mar-09	29.2	33.19	1.122	62.9
Abr-09	29.7	35.51	2.321	65.5
May-09	30.0	37.29	1.783	67.6
Jun-09	30.3	37.95	0.660	68.6
Jul-09	30.7	40.11	2.161	71.1
Ago-09	31.0	40.81	0.695	71.9
Sep-09	31.1	43.69	2.884	74.9

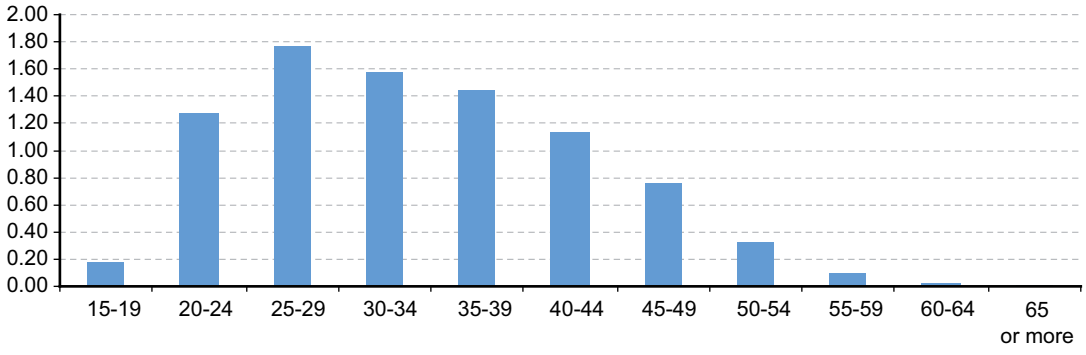
Source: Superintendencia Financiera de Colombia (Financial Superintendency of Colombia)
Calculations BBVA Research

Although in the short term the funds may show a significant volatility in their returns, as shown by the period of financial crisis in 2008, in the long term, this will tend to be more limited. The performance of the funds should be assessed from this perspective.

Investment in pension funds should undoubtedly be from the perspective of a long-term investment horizon, given the age structure of their members (see Chart 3).

Chart 3

Number of members by age group: 2009 (Number of members)



Source: Financial Superintendency of Colombia BBVA Research

In fact, those forming part of the individual capitalization system were concentrated in lower age ranges, with 72.4% of them under the age of 39. In other words, most of the members have to remain in the system for at least 18 years or more. At the other end of the scale, only 5.8% of the members were over 50 years of age.

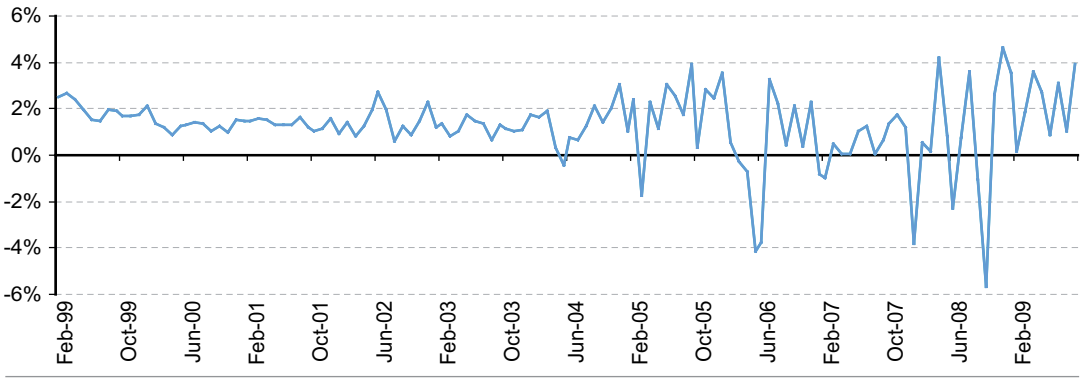
B) Returns on funds from a long-term perspective

1. Historical perspective

Between January 1999 and September 2009, the nominal annual average return of the funds was 15.2% and their volatility was 5.2%. The real average annual return was 9.1%. The behavior of the returns can be broken down into two periods: one runs from January 1999 to mid-2004, and the other from mid-2004 to September 2009, with a clear change in volatility presented by the series, as is shown in Chart 4.

Chart 4

Monthly return of pension funds (2001-2009)

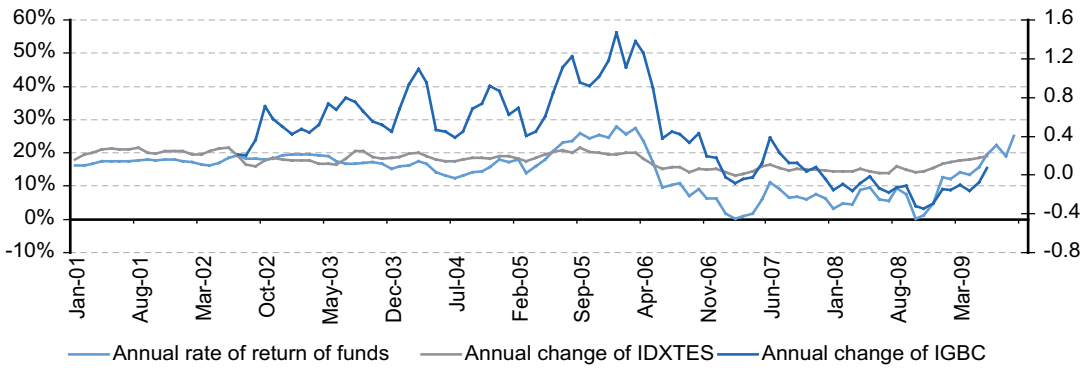


Source: Financial Superintendency

In the first period, the annual volatility was 1.8%, a far lower level than the 7.1% in the second period. The annual average return of the funds stood at 16.9% in the first period and 13.3% in the second. The high volatility in the second period can probably be initially explained by the fluctuations in the TES bonds, with the TES rate for September 2014 falling 46% in only 14 months (November 2004 to February 2006) and a subsequent increase of 50.1% in only 4 months (February to June 2006). Subsequently, the greater volatility was probably the result of the greater proportion of equity within the fund portfolio[5] (see Chart 5).

Chart 5

Annual change in the IDXTES and IGBC and annual fund rates of return

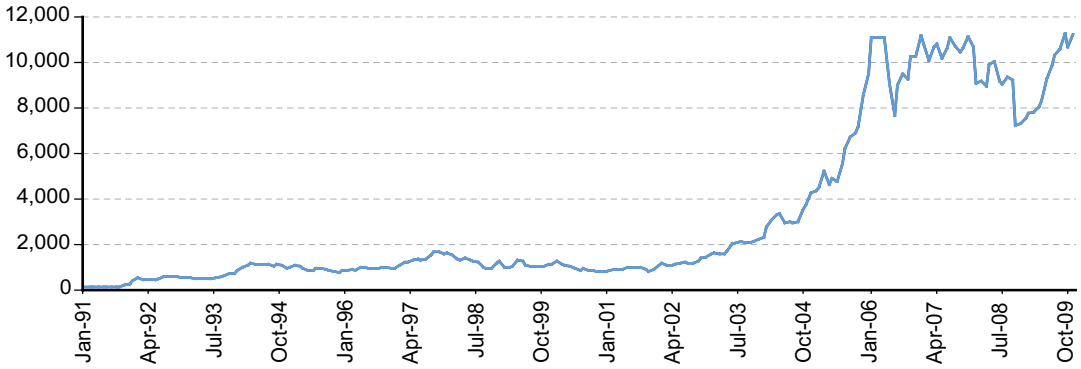


Source: Bank of the Republic and Colombian Stock Market. BBVA Research

If we analyze the IGBC[6] over a longer period (1991-2009), we see that there was a steady rise over the period, with an annual average growth of 24.3% and a volatility of 30.5%. The series present two trends in the period under analysis. The first starting in 1991 and lasting until November 2005, with a rate of average annual growth of 29% and an annual volatility of 31.1% (see Chart 6).

5: Between June 2004 and September 2009 the IGBC had an annual volatility of 30.7%.
6: In 2001, the stock exchanges of Medellin, Cali and Bogota were merged into the Colombia Stock Exchange. The overlap period of the IGBC index for before the merger of the three stock exchanges was calculated by applying the growth rates of the index of the Bogota Stock Exchange, given that it had the highest trading volumes.

Chart 6:
Colombian Stock Exchange Index



Source: Colombia Stock Change and BBVA Research

In the second period (December 2005 to November 2009), the annual average rate of growth was 13.6% and its volatility was an annual 30.6%. In other words, in the first period under analysis, the series presented an exponential growth which eased until the start of the second period, where, although the level of volatility was similar, the movements in the IGBC were within the band of 7,200 and 11,258.

2. Future outlook[7]

Although historical experience may offer signs of possible trends in investment matters, it is important to bear in mind that these signs do not represent a guarantee of any kind on possible future performance. Therefore, for forward assessment of fund investments we have to take into account a variety of different scenarios covering the possible price performance of the different classes of assets that make up the fund portfolios under the forecast investment rules.

One way of presenting possible scenarios of the future performance of financial asset prices is to carry out simulation exercises. Therefore, in this work we use the Monte Carlo simulation technique to project the prices of the main fund asset classes, fixed-income and equity, within a range of investment horizons from 1 to 50 years and using the random walk theory that explains the behavior of these financial series[8].

a) Model for the long-term dynamics of financial asset prices

The prices of the main asset classes, fixed income and equity, are modeled as random variables using a multiplicative model with the following general form:[9]

Formula 1

$$P_t = P_0 e^{gT}$$

The model indicates that the price of a financial asset at time t = T is equal to the price of the asset at time t = 0 increased exponentially at a rate “g” over a T-year horizon.

As a result, the behavior of the price of the asset depends on the behavior of “g”. One widely used hypothesis in the financial sector for the possible behavior of “g” is that it behaves as a random variable (rv) with a normal probability distribution and constant mean and variance.

7: This section is based on the working document of Herrera (2010) for the case of Mexico.
8: The Monte Carlo simulation is an algorithm that carries out repeated random sampling of securities that is then used as the input in a performance equation for a variable of interest.
9: An alternative way of specifying the asset price model would be additive. However, a specification of this type would not lead to a lognormal distribution for asset prices which, as mentioned below, enables us to capture some relevant characteristics. For more details of these alternative specifications and their limitations, see Luenberger (1998).

The relevant point about "g" being distributed as a random variable with a normal probability distribution is that statistically it is possible to demonstrate that the time series on which its return is represented, which for the case analyzed here is the asset price of equity and the discount rate of the fixed-income assets, behaves as a random variable, but with a lognormal probability distribution. This lognormal distribution makes it possible to capture at least three characteristics of financial asset prices:

1. Prices are always positive
2. At any point in time, prices are uncertain since they are affected by the variance of "g". However, when the variance has the value "zero", there is a deterministic model for the price of a fixed-income asset, where the interest rate is determined a priori for a particular term, as occurs in the case of "zero coupon" bonds.
3. In short timescales, price changes are continuous.

In the multiplicative model, the value of "g" is obtained by applying logarithms on both sides of the equation:

Formula 2

$$\ln(P_t)=\ln(P_0)+gT$$

Formula 3

$$\ln\left(\frac{P_t}{P_0}\right)=+gT$$

Formula 4

$$g=\frac{1}{T}\ln\left(\frac{P_t}{P_0}\right)$$

The rate "g" is therefore a rate of return annualized over a time horizon from 0 to T. In this context, "gT" may be interpreted as an accumulated growth rate which also has a normal probability distribution.

A widely-used hypothesis in the financial sector is that "gT" follows a stochastic behavior described by a geometric Brownian movement (GBM) or the Wiener "dzt" process (See Hull (2008)). Under this hypothesis, any random variable "x" exhibits a dynamic over time given by a stochastic differential equation of the type:

Formula 5

$$dx_t=vdt+\sigma dz_t$$

Where:

Formula 6

$$dz_t=\varepsilon_t\sqrt{dt}$$

With

Formula 7

$$\varepsilon_t \approx N(0,1)$$

This stochastic equation has an analytical solution given by the equation:

Formula 8

$$x_t=vt+\sigma z_t$$

Therefore, under the GBM hypothesis for "gT", prices would behave as follows:

Formula 9

$$P_T=P_0 e^{vt+\sigma dx}$$

Where “gT” is distributed as a normal random variable with a constant average and variance:

Formula 10

$$gT \approx N(vT, \sigma^2 T)$$

The change over time in the asset price is as follows:

Formula 11

$$\ln\left(\frac{P_t}{P_0}\right) = vt + \sigma dz_t$$

Formula 12

$$d\ln(P_t) = vt + \sigma dz_t$$

This behavior could be expressed equivalently in terms of P(t) as follows:

Formula 13

$$\frac{dP_t}{P_t} = \mu dt + \sigma dz_t$$

Where

Formula 14

$$\mu = v + \frac{1}{2}\sigma^2$$

Following Luenberger (1998), the above stochastic process for the price of a financial asset may in turn be extended to the case of the value of a portfolio with n assets, in such a way that the price of the i-th asset where i=1, 2, 3,…n is given by a behavioral equation as follows:

Formula 15

$$\frac{dP_i}{P_i} = \mu_i dt + \sigma dz_i$$

With covariance

Formula 16

$$Cov(dz_i, dz_j) = \sigma_{ij} dt$$

Therefore, the change in price for each asset i, at an instant of time t, has a lognormal probability distribution with an expected value and variance, as given by the following two equations:

Formula 17

$$E\left[\ln\left[\frac{dP_i(t)}{P_i(0)}\right]\right] = vt = (\mu_i - \frac{1}{2}\sigma^2)t$$

Formula 18

$$Var\left[\ln\left(\frac{dP_i(t)}{P_i(0)}\right)\right] = \sigma_i^2 t$$

A portfolio with “n” assets is built by assigning a weight w(i) to each asset i=1, 2, 3,…n where the sum of all the weights w(i) is equal to 1. As a result, the instantaneous rate of change of a value in a portfolio V is given by the equation:

Formula 19

$$\frac{dV}{V} = \sum_{i=1}^{n} w_i \frac{dP_i}{P_i} = \sum_{i=1}^{n} w_i \mu_i d_t + w_i \sigma dz_t$$

Where the variance in the stochastic term dz(t) is given by the term:

Formula 20

$$E(\sum_{i=1}^{n} w_i dz_t)^2 = E(\sum_{i=1}^{n} w_i dz_i) E(\sum_{j=1}^{n} w_i dz_j) = \sum_{i,i=1}^{n} w_i w_j \sigma_{ij} d_t$$

Therefore, for a lognormal portfolio V(t), the expected value of its return and its variance are given by the following equations:

Formula 21

$$E\left[ln\left(\frac{dv}{V}\right)\right] = vt = \sum_{i=1}^{n} w_i \mu_i t - \frac{1}{2} \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} t$$

and,

Formula 22

$$\sigma^2(t) = \sum_{i,i=1}^{n} w_i w_j \sigma_{ij} d_t$$

Where “v” gives the annualized growth rate of the portfolio’s value and is a function of the assignment of assets through the w(i).

Formula 23

$$V = \frac{1}{t} E\left[ln\left(\frac{dV}{V}\right)\right]$$

In this note, we estimate the value of “v” for 250 scenarios of portfolio simulation composed of a fixed-income asset and an equity asset in different time horizons that range from 1 to 50 years.

In order to model the behavior of returns of fixed-income instruments in the pension fund portfolios (AFPs) an index was also constructed of interest rates with different maturities. This was weighted by the participation that each of the instruments currently has in the pension system. In order to carry out the fixed-income simulation exercises, a working hypothesis was used in which the prices of these assets are proportional to the prices of short-term instruments, and all the volatility in the prices comes from the volatility of the short-term instruments.

A functional form which is compatible with the above is given by the Ornstein-Uhlenbeck behavior equation for short-term rates r(t) cited by Vasicek (1977), which is specified as:

Formula 24

$$dr = \alpha(Y-\gamma)dt + \sigma dz \text{ with } a>0$$

This equation contrasts with a Wiener process such as those explained above, in that it defines a stationary behavior for the random variable. As a result, in this equation the term “ (γ-r)” represents a force which takes the process towards gamma, its average long-term value. The value of alpha is known as the velocity of regression to the mean.

Vasicek demonstrates that it is possible to construct an interest-rate curve for different terms based on the this equation by calculating prices for “zero coupon” bonds using equations which are only dependent on the “alpha” and “gamma” parameters.

Vasicek’s starting point is that the performance of any bond at a time t and with maturity at T is given by an internal rate of return at t, which is an inverse function of its price.

Formula 25

$$R(t,T)=-\frac{1}{T}\ln(P(t,t+T)) \text{ with } T>0$$

Based on the above, the short-term interest rate is defined as an instantaneous rate when t tends to zero.

Formula 26

$$r(t)=r(t,0)=\lim_{T\to 0} R(t,T)$$

Vasicek demonstrates that the price of a bond with a maturity T is given by a specific functional form:

Formula 27

$$P(t,T,\gamma)=\exp\left[\frac{1}{\alpha}(1-e^{-\alpha(T-t)})(R(\infty)-\gamma)-(T-t)R(\infty)-\frac{\sigma^2}{4\alpha^3}(1-e^{-\alpha(T-t)})^2\right] \text{ with } t\leq T$$

Where, R(∞) represents the yield to maturity of a bond in the very long term (when T tends to infinity).

Formula 28

$$R(\infty)=\gamma+\frac{\sigma}{\alpha}-\frac{\frac{1}{2}\sigma^2}{\alpha^2}$$

Based on these equations, Vasicek demonstrated that the interest rate structure for different terms can be calculated using the equation:

Formula 29

$$R(t,T,)=R(\infty)+(r(t)-R(\infty))\frac{1}{\alpha T}(1-e^{-\alpha T})+\frac{\sigma^2}{4\alpha^3 T}(1-e^{-\alpha T})^2 \text{ with } T\geq 0$$

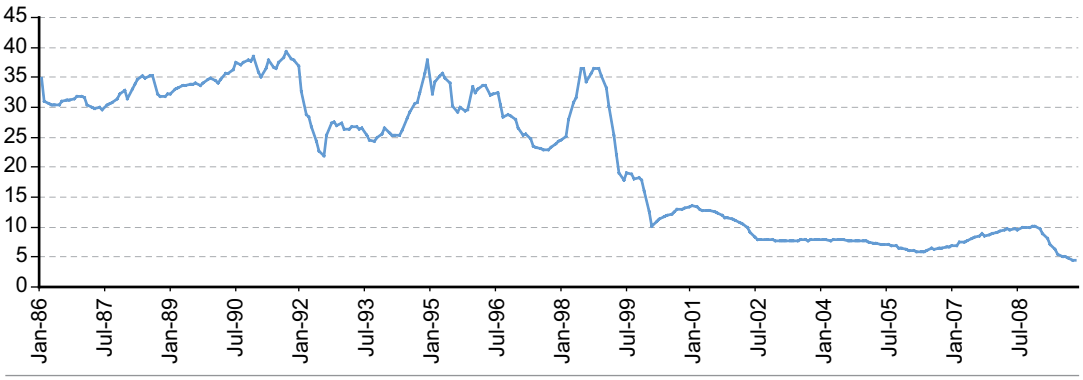
Finally, it is important to note that for the purpose of applying the above models to the Colombian experience, the interest rates of the 90-day term deposit certificate (CDT) were used as representative of short-term securities. The CDT is not a market rate, but constitutes a leading interest rate. The Global Colombian Stock Market Index (IGBC) was also taken as a representative of equity.

The calculation of the respective parameters for modeling the classes of assets explained above took as a reference the monthly series of 90-day CDT for the period January 2002 to November 2009 and the IGBC for January 1991 to November 2009. It should be noted that the fixed-term deposits DTF were taken as the representative rate for the Colombian economy because they are able to provide a longer-term series. This is despite the flaws this may present compared with other rates such as the bank reference rate IBR. It was decided to take 2002-2009 as the reference period for the DTF, as it was considered that the rate behaved in a stable fashion during this time. The historical behavior of the DTF is analyzed below.

In the period between January 1986 and November 2009 the nominal DTF fell from 34.8% to 4.4%, with a volatility for the period of 21.4% (see Chart 7).

Chart 7

Short-term leading interest rate for the period 1986-2009 DTF, average monthly return, annual %



Source: Bank of the Republic and BBVA Research

Between 1986 and 1992 the DTF was an average of 34% and its volatility stood at 2.7% (9.2% annual). With the introduction of Central Bank autonomy and its mandate to control inflation, the DTF was substantially reduced to an average of 28% between 1992 and mid-1998, with an annual volatility of between 4%-13.8%). From mid-1998 until the start of 2000, the rate was reduced to a third from 36.6% to 10.2% (in real terms, the DTF was around 0.5%), with a volatility of 9.2%. This can be explained basically by the crisis suffered by the economy during this period, due to the contraction of the financial sector and a looser monetary policy by the Bank of the Republic[10]. Since then, the DTF has fallen steadily to a level of 4.4% in November 2009, with a volatility of around 7%. Based on the analysis of the data it was considered that the nominal long-term DTF rate would be around 8%. From 2002, the rate has stabilized, reaching an average of 7.75% between January 2002 and November 2009. Volatility was at an annual 4.9%.

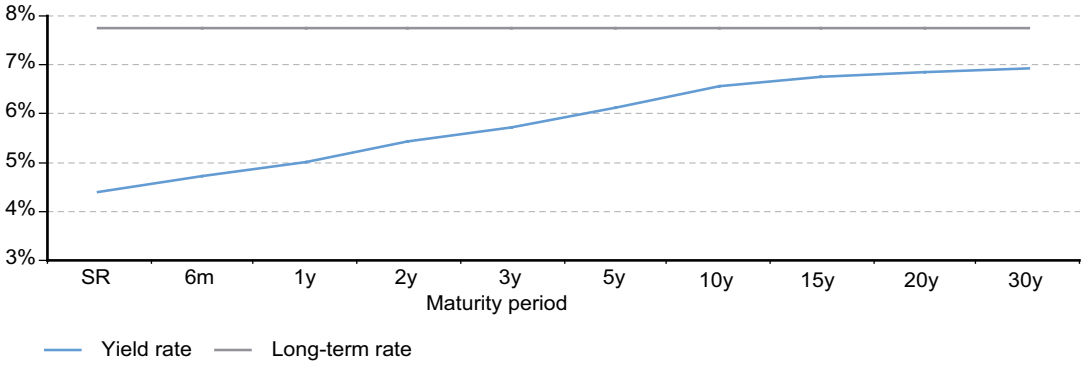
Long-term inflation was assumed to be 3% estimated by the Colombia BBVA Research. In the case of equity, an annual historic return of 30.5% and volatility of 24.3% were assumed (values observed for the period 1991-2009).

b) Results of the simulations

For fixed income assets, the methodology used by Vasicek makes it possible to simulate the behavior over time of short-term interest rates (DTF), and to use this behavior to estimate an interest rate or performance curve for each point in time.

Chart 8:

Fixed-income spot curve



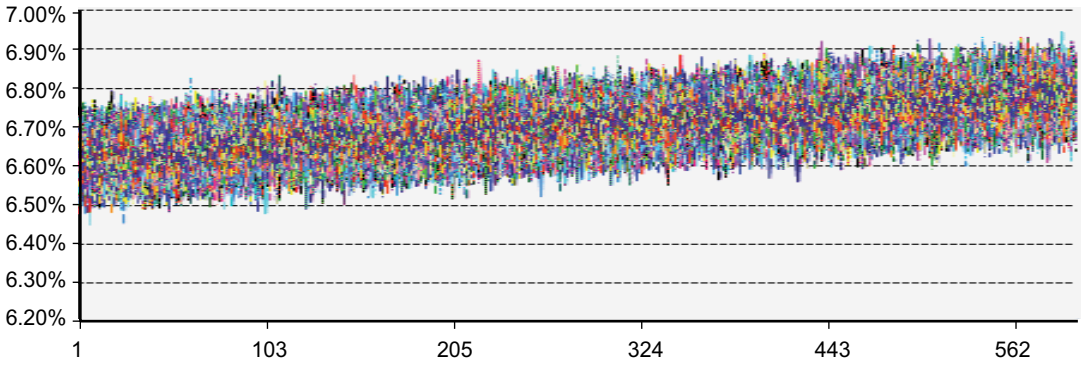
Source: BBVA Research

10: Corporación Financiera del Valle (2001).

The simulations of nominal fixed-income return, which have been weighted based on the composition according to the maturity of fixed-income in the portfolios of funds in 2009, show a rising trend, with an average from 6.6% to 6.7% at 50 years (see Chart 9). This return is fairly limited, with an average annual volatility of 0.04%.

Chart 9

Forecast for fixed income in a 50-year horizon (250 paths)

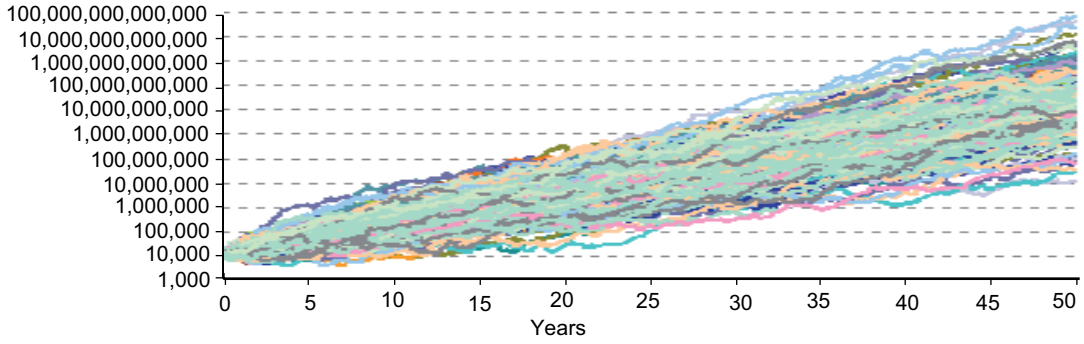


Source: BBVA Research

The simulations show a fairly substantial increase in equity prices (see Chart 10). This is to be expected, given the assumed base for the simulations of average annual growth of 24.3%.

Chart 10

Forecast of equity in a 50-year horizon (250 paths)



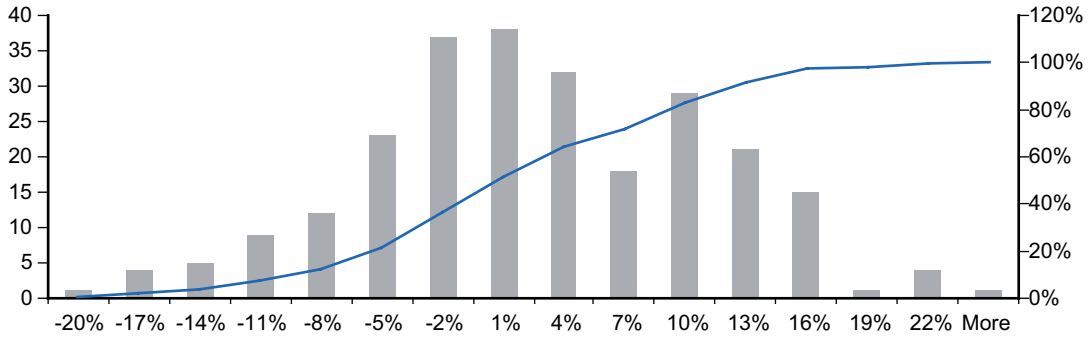
BBVASource: BBVA ResearchResearch

Equally, although the monthly return from equity assets is fairly volatile, of around 30%[11] annually, the accumulated returns are limited around one value in longer time horizons. In fact, if we examine equity volatility, for the 250 projected paths, in any particular month, the volatility can be seen to be fairly high, with values of returns at between -20% and over 20% and a mode of 1% (see Chart 11).

11: The result of calculating the average volatility of all the equity paths for a period of 50 years

Chart 11

Distribution of returns, month T = t (% frequency and accumulated frequency).

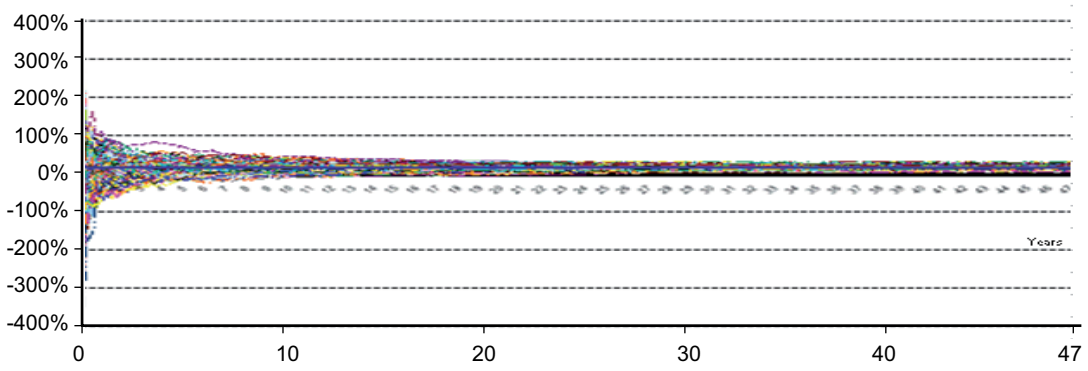


Source: BBVA Research

In addition, the return offered by an equity instrument is located within a range of values when the investment horizon is considered explicitly in the calculation of return, through a compound annual rate of growth between the start and finish periods of the investment. The longer the investment horizon, the lower appears to be the dispersion between the possible returns (see Chart 12).

Chart 12

Real accumulated return on equity in each time horizon, 0 to 47 years (annualized rate of monthly growth)



Source: BBVA Research

Although in the short term a pension fund member with a portfolio composed mainly of equity assets may experience a high volatility, in the long term this volatility reduces, and the return is restricted around a certain value, which is equivalent to the statistical long-term expectation. In this way, maintaining a high proportion of equity assets is justified by the greater returns from these kinds of assets for members whose investment horizon is long, as may be the case of young people whose resources can be maintained in the funds for a significant number of years.

This study includes different portfolio simulations with different combinations of equity and fixed income in order to analyze the long-term results. This allows us to appreciate the different profiles that can describe the returns and volatilities over time, in accordance with the composition of the portfolios and the changes that may be produced at the different stages of the members' working life. The simulation considers 250 scenarios for different portfolios.

b.1 Mexican life-cycle case

The first simulation for the case of Colombia applies the Mexican **life-cycle system.** In this scenario, members between the ages of 18 and 26 are in a fund where equity accounts for 30% of the total portfolio. Between the ages of 27 and 36, they move to another fund in which equity accounts for 25%. As they grow older, their assets are moved to funds with a lower proportion of equity (see Table 3).

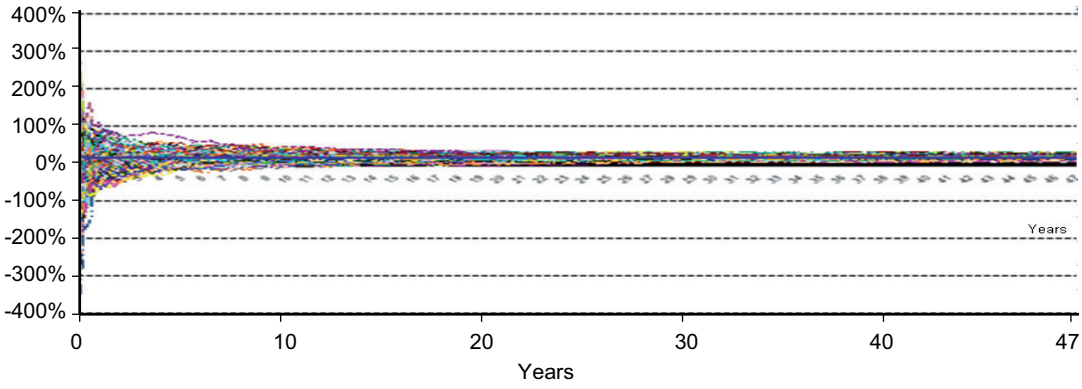
Table 3
Composition of portfolios by members' ages: the case of Mexico

	Siefore 5	Siefore 4	Siefore 3	Siefore 2	Siefore 1
Precentage of Equity in Fund	30%	25%	20%	15%	0%
Minimum age	18	27	37	46	56
Maximum age	26	36	45	55	65

Source: BBVA Research

In this scenario, the real annual average accumulated return after 39 years is **7.2%** and volatility **1.1%** (in the case of a woman who has deposited one peso at 18 years of age in the pension system and has kept it there until the age of 57). If the person remains in the system for 44 years (in the case of a man who deposits one peso at the age of 18 in the pension system and keeps it there until he is 62) he will obtain a real annual average return that is slightly lower, at **6.8%** and also a lower volatility, of **1%** (see Chart 10). It should also be noted that the reduction of the volatility in average annual returns, which (depending on the model), vary between 9.7% for the first year to the rates mentioned above of around 1% at 39 and 44 years. In the case of Mexico, the volatility is reduced significantly over time by the introduction of a greater proportion of fixed-income into the portfolio as it becomes older, with the proportion reaching 100% after 56 years of age (see Chart 13).

Chart 13
Simulation of Mexican multi-funds for Colombia



Source: BBVA Research

b.2. Different combinations of equity and fixed-income and multi-funds (Table 4)
Currently, the portfolio of the funds in Colombia is composed of approximately 40% equity and 60% fixed income. However, it should be noted that the proportion of fixed-income used to be even higher, at 77.2% of the total portfolio in 2006, and historically it represented a greater proportion.

Below we present the results of simulating a scenario that can be considered as an average for the Colombian case, which maintains a composition of 30% in equity and 70% in fixed income (**E30%-FI70%**- see Table 4). A fund with this composition would yield, depending on the model, a real average of 8.6% over 39 years [12].

12: It is worth mentioning that the annual real average return of the funds was at 10.03% over the last 12 years (February 1998 – February 2010). The real average return in the last 3 years (March 2007-February 2010) was 21.97%.

Table 4

Average annual returns and volatility for different time periods

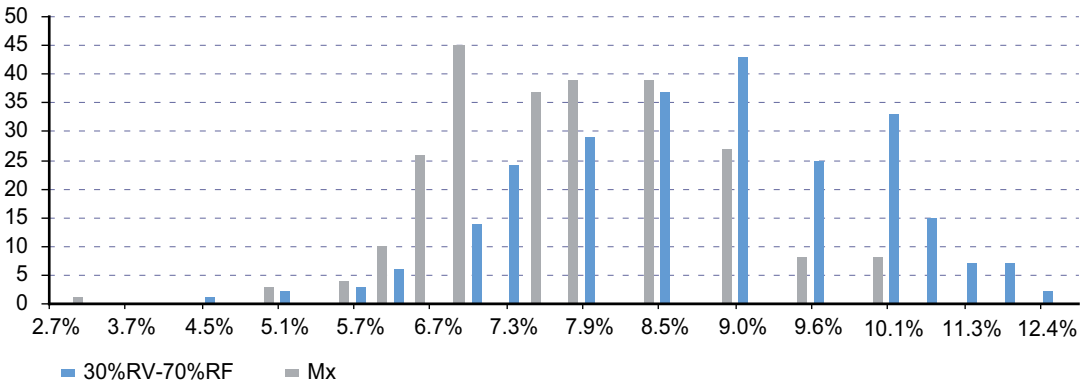
	BASE SCENARIO: E30		E 50		EV80		Mexican Life Cycle	
Año	Average annual return	Volatility	Average annual return	Volatility	Average annual return	Volatility	Average annual return	Volatility
1	8.56%	9.71%	11.41%	16.14%	15.00%	25.79%	8.56%	9.71%
5	8.27%	4.02%	10.94%	6.66%	14.24%	10.63%	8.27%	4.02%
10	8.33%	3.02%	11.02%	5.00%	14.35%	7.97%	8.25%	2.97%
39	8.57%	1.46%	11.40%	2.40%	14.94%	3.83%	7.24%	1.13%
44	8.59%	1.36%	11.42%	2.22%	14.97%	3.54%	6.84%	1.02%

Source: BBVA Research

If we compare these results with the case of Mexico we can see (as was to be expected) that, until the fifth year, both the accumulated return and its volatility are equal to the extent that the portfolios of the two simulations are the same (see Table 4). At the tenth year the differences begin to emerge, with an accumulated average return that is greater in each case **30%E- 70%FI** which, in the case of Mexico, is where from the ninth year the proportion of equity in the portfolio is reduced to 25% and finally to 0% in the last years before the compulsory requirement age. After 39 years, the difference in returns is at 33 basis points and after 44 years it is 175 basis points. The volatility is 22.5% and 25% lower in the Mexican case than in the case of **30%E-70%FI** at 39 and 44 years, respectively (see Table 4). It is interesting to observe how the life-cycle system limits returns still more. In the Mexican case, 85.2% of returns are within the range of values of 5.7% and 8.8%, with 18% of them at the value of 6.7% (see Chart 7), while in the **30%E- 70%FI** case, 90% of the observations are within the range of values of 6.2% and 10.7%, with 17.2% at a value of 9%. In other words, although the returns are higher in the **30%E- 70%FI** case, they are also less limited than in the Mexican case (see Chart 14).

Chart 14

Distribution of returns in the Mexican and 30%E- 70%FI case (frequency)



Source: BBVA Research

This simulation shows that, through the implementation of the multi-funds, pension fund members can obtain higher returns than with the single fund system, depending on the proportion of equity included in the funds. In carrying out the simulation in the Mexican case, whose riskiest fund contains E30% compared with the case of the single fund with E30%, the results obtained show lower returns in the Mexican case. This simulation shows that the life-cycle system limits returns still more. Due to the above, and with the aim of increasing returns by the introduction of the multi-funds, the riskiest fund introduced in Colombia should have a higher proportion of equity than the current single fund.

What would be interesting for the Colombian case would be to have as the riskiest fund one in which the proportion of equity was greater than the intermediate scenario simulated above **(30%E- 70%FI)** and greater than the current single fund **(40%E).** To this extent, the risky fund could come closer in composition to the Chilean and Peruvian cases with a composition of 80%E, for example. In these circumstances, as can be seen in Table 4, with a composition of 80% equity and 20% fixed-income **(80%E - 20%FI),** one peso invested for 39 years would have a real annual average return **near 15%.**

This return stands at **11.4%** for a portfolio composed of **50%E - 50%FI** (see Table 4). In the case of **50%E - 50%FI,** 85% of the returns are between 8.9% and 14.3%, with 19% of the observations giving a value of 12.2%. In the case of **80%E - 20%FI,** 88% of the returns are between 9.9% and 20.5%, with 17.2% of the observations giving a value of 14.4%[13].

We will now analyze the impact of introducing a convergence rule for these riskier funds mentioned above. This specifies that there should be a move towards portfolios with a greater proportion of fixed-income in the final years of saving accumulation, in order to limit the risk for people who are close to retirement age. The convergence rule introduced means that 5 years before retirement age (at 34/39 years of contributions to the system in the case of men/women, respectively) each member should have a minimum of 20% of his/her portfolio in a conservative fund. This percentage increases by 20% each year with the increased age of the member. After 5 years, the member will have a portfolio which will be 100% in the conservative fund. For the purpose of this exercise a conservative fund is considered to be one with a proportion of 10% of equity. Thus at the end of the period of accumulation, the member will have a portfolio with **10%E.**

Table 5

Return and volatility obtained with a composition of the 80%E portfolio with and without the convergence rule

	Scenario 1: E80% no convergence		Scenario 2:EV80% with convergence			
			Female		Male	
Año	Average annual return	Volatility	Average annual return	Volatility	Average annual return	Volatility
39.0	14.94%	3.83%		3.59%	14.94%	3.83%
44.0	14.97%	3.54%			14.31%	3.43%

Source: BBVA Research

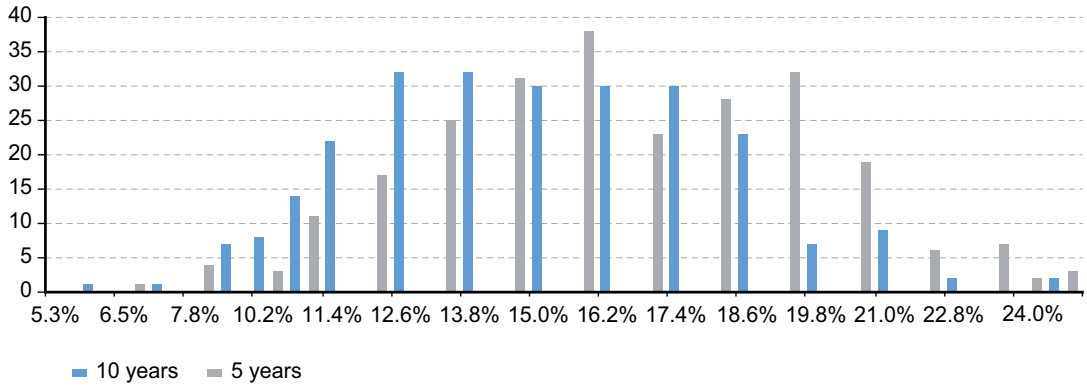
The convergence rule reduces volatility. The above can be seen by comparing the **80%E portfolios without convergence** with the **80%E portfolios with convergence** [14] (scenarios 1 and 2 in Table 5). From the moment at which the convergence rule kicks in (at 34 years in the fund in the case of women and 39 years in the case of men), the volatilities of the scenario without convergence exceed those of the scenario with convergence. This difference increases as convergence advances. The same happens with returns: it is greater in the case of non-convergence than in the case of convergence and this difference increases as the convergence process progresses. This is to be expected in that the rule aims to protect the savings of people who are close to retirement by exposing them to fewer changes in returns, although they receive lower accumulated average returns as a result. In the case of **E80% with convergence** with retirement after 39 years of age, the model shows a greater relation between returns and volatility than in the case of **E80% without convergence** (see Table 5). If the same rule of convergence simulated above is carried out starting at an earlier age, for example 10 years before retirement age and finishing 5 years before it, the volatility of the accumulated average return obtained at the time of retirement would be lower but so would the accumulated return (see Chart 14a).

13: It is worth stressing that high proportions of equity give rise to portfolios with very high returns, as a result of initial assumptions of high average returns used at the time of projecting equity prices. The model is biased towards the introduction of higher proportions of equity, but it is very useful for studying the direction of the changes produced in both returns and in volatilities when different portfolios are introduced.

14: In the scenario in which the person maintains a portfolio with E80% throughout the period of accumulation, the divergence in real returns is between 10.1% and 22% after 39 years. In the case of E80% with convergence, 85% of returns are between 9% and 19.1%. It can thus be seen that the convergence rule really reduces the range of returns that can be obtained.

Chart 14a

Distribution of the return in the case of 80%E with convergence 5-years earlier and 80%E with convergence 10 years earlier (frequency)



Source: BBVA Research

The accumulated average return of the scenario with convergence 10 years earlier is at 12.94% at 39 years, compared with 14.16% in the case of the scenario with convergence 5 years earlier. In other words, it marks a reduction in return of 122 basis points. It should be pointed out that in the case of convergence 10 years before retirement the relationship between the real accumulated average annual return and volatility is lower than in the convergence 5 years before retirement, showing a lower return per unit of risk in the first case (see Table 5a).

Table 5a

Return and volatility obtained with an 80%E portfolio and the convergence rule 5 years and 10 years before retirement

		E80%	E80% 5-year convergence	E80% 10-year convergence
At 44 years	Yield	14.97	14.31	13.18
	Volatility	3.54	3.43	3.18
	Sharpe	4.23	4.17	4.14
At 39 years	Yield	14.94	14.16	12.94
	Volatility	3.83	3.59	3.42
	Sharpe	3.90	3.94	3.78

Source: BBVA Research

In these circumstances it could be alleged that the scenario convergence 10 years before retirement, as simulated above, would be lower than that of convergence 5 years before.

Of the simulations carried out until now, it could be alleged, **first**, that given that convergence reduces accumulated average return at the time of retirement, it could be expected that for the case of Colombia, not only would the risky fund have a greater proportion of equity than the single current fund, but that even the moderate fund would have a greater proportion of equity than the current fund. Even more so if we consider that the current fund is probably the default fund. In fact, international experience shows that most people do not choose their fund, and so the design of the default fund is of great importance. In the case of Colombia, the law stipulates that the default fund is the moderate or conservative portfolio and thus the moderate fund will be chosen. To sum up, if the moderate fund with the proportion of equity of the current fund is maintained, this would presumably imply that the implementation of the multi-funds would by itself reduce the amount of equity in the system, as older people would enter directly with their assets in the conservative fund, whose composition in equity is lower than the current fund, and thus the average return would be reduced. Second, when an alternative of convergence 10 years before the retirement age is considered for the riskiest fund (E80%), it was established that this scenario would be worse in terms of returns and Sharpe ratio than the above scenario with a transition 5 years before retirement. However, because this convergence rule is also applied to the moderate fund, it should be assessed for this fund as well. The convergence rule should be assessed in terms of the return/volatility ratio, and always on the basis of a target return for each fund.

The following is a simulation of a possible scenario for the case of Colombia: a risky fund with a proportion of E80%, a moderate fund with a proportion of E45% and a conservative fund with a proportion of E10%. It should be noted that the moderate fund would have a proportion of equity that is slightly higher than the current single fund. In addition, the 5-year convergence before retirement would be implemented by Law. We have simulated this above. We also simulated the case of a member who chooses a risky fund for the first years of accumulation and a moderate fund at an intermediate age.

b.3. Scenario E80%-E45% with convergence (Tables 6 and 7).
In this scenario a member can choose a fund with greater risk between 18 and 36 years of age, moderate risk between 37 and 51/57 years of age (women/men) and is subject to a convergence rule starting at 52/57 years of age (women/men) (see Table 6).

Table 6

Composition of the portfolio of a woman/man who chooses the three funds during her/his life and is subject to the convergence rule

	Higher risk fund	Moderate risk fund	moderate and conservative fund	
Percentage of equity in the fund	80%	45%	Convergence Rule	
Maximum age	18	37	52/57	F/M
Minimum age	36	51	57/62	F/M

Source: BBVA Research

In these circumstances, the real average return obtained for the period of 39 years (women) would be **12.2%**, with an annual volatility of **3%** [15] (see Table 7, columns 3 and 4). In the case of men, in scenario 1: **E80%-E45%, no convergence rule**, after 44 years the return obtained would be **12.1%**, with a volatility of **2.8%** for the period (see Table 8, columns 4 and 5). In this scenario, both in the case of women and men, a greater ratio is obtained between the annual average accumulated return and its volatility than would be obtained in the case of **E80% with convergence**. It may be thought that although the return in the scenario E80%-E45%, no convergence rule, is lower than the E80% with convergence scenario, it is still high and with a greater average rate of return per risk unit (4.1 compared with 3.9 after 39 years and 4.4 compared with 4.2 after 44 years) (see Table 7).

Table 7

Returns and volatility obtained with a composition of the E80%-E45% portfolio and the convergence rule, E80% without the convergence rule and E80% with the convergence rule

	Scenario 1: E80% E45% and Convergence Rule				Scenario 2: E80% no convergence		Scenario 3: E80% with convergence			
	Female		Male				Female		Male	
Year	Annual average returns	Volatility	Annual average returns	Volatility	Annual average returns	Volatility	Annual average returns	Volatility	Annual average returns	Volatility
39.0	12.23%	3.01%	12.67%	3.10%	14.94%	3.83%	14.16%	3.59%	14.94%	3.83%
44.0			12.09%	2.76%	14.97%	3.54%			14.31%	3.43%

Source: BBVA Research

15: In this scenario, 88% of the real returns obtained on the 250 different paths are between 8.3% and 18%, and 17% of them are 10.5%.

Given that the convergence rule is applied in the case of men five years after the case of women, scenario 1 **-E80%-E45% with convergence-** for the cases of men and women begin to differ from year 34 (the time when convergence begins for women). In fact, the real accumulated average annual return at year 35 in the case of women is 12.8%, while for men it is 12.83%. The return for men remains above that for women until year 39, when the woman retires. The above is due to the fact that the convergence rule in the case of women is at a more advanced stage than in the case of men. In addition, it should be added that the convergence rule reduces volatility. In fact, volatility at 39 years in the system is at 3.01% in the case of women and 3.10% in the case of men with the same number of years (see Table 7). Let us now suppose a scenario in which the member does not chose and is assigned a fund by default.

b.3. Default scenario: E45% with convergence (Table 8).
It is worth mentioning the importance that the moderate fund has in the Colombian case for the success of the multi-funds, because this fund becomes the default fund. If the sole fund is maintained at the current proportion **(E40% without convergence)** the real accumulated average annual return after 39 years is 10%. The portfolio of **E45% without convergence** would give a real accumulated average annual return of 10.73% after 39 years. In the case of the portfolio of **E45% with convergence,** the accumulated return (10.23% real annual average) would be in an intermediate position between **E40% without convergence** and **E45% without convergence.** In this scenario the return would be intermediate, as would the volatility (see Table 8 and Chart 15).

Table 8

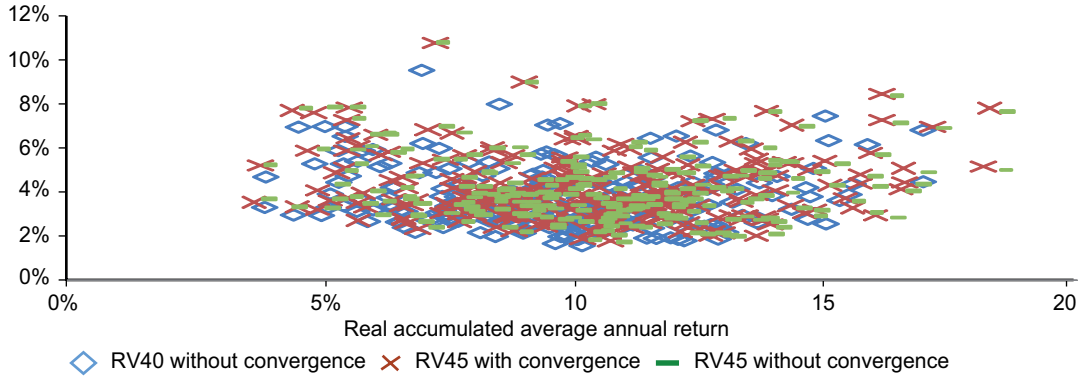
Return and volatility obtained with portfolios with a proportion of E40% and E45% without the convergence rule and E45% with the convergence rule

	Scenario 1: E40% no convergence		Scenario 1 1: E45% no convergence		Scenario 2: E45% with convergence			
					Female		Male	
Year	Annual average returns	Volatility	Annual average returns	Volatility	Annual average returns	Volatility	Annual average returns	Volatility
39	10.03%	1.93%	10.73%	2.16%	10.28%	2.04%	10.73%	2.16%
44	10.05%	1.79%	10.75%	2.00%			10.37%	1.94%

Source: BBVA Research

Chart 15

Return and volatility obtained with a portfolio of E40% and E45% without the convergence rule and E45% with the convergence rule



Source: BBVA Research

At the time of choosing the default portfolio we should compare both the returns and the volatility and the relationship between these two variables. Based on the simulations it can clearly be established, for example, that convergence 10 years before retirement does not represent an option (see Table 8a).

Table 8a

Return, volatility and Sharpe Ratio

		E40%	E45%	E45% convergence 5 years	E45% convergence 10 years
At 44 years	Return	10.05	10.75	10.37	9.74
	Volatility	1.79	2.00	1.94	1.81
	Sharpe	5.61	5.38	5.35	5.35
At 39 years	Return	10.03	10.73	10.28	9.60
	Volatility	1.93	2.16	2.04	1.95
	Sharpe	5.20	4.97	5.04	4.92

Fuente: BBVA Research

In fact, the **E40%** option presents a higher return, lower volatility and a higher Sharpe ratio than the **E45% convergence 10 years before retirement** (at both 39 years and at 44 years). The options **E45%** and **E45% with convergence 5 years before retirement** present higher average returns than the **E40%** option, despite its volatilities being greater and its Sharpe ratios lower. As the returns of the simulated **E45%** scenarios are greater, we should assess which of the two should be chosen. The return in the **E45%** scenario is 3.7% higher than in the **E45% scenario with convergence 5 years before retirement** and volatility is 3.1% higher.

To sum up, the moderate portfolio should probably have a greater proportion of equity in the Colombian case than the current single fund to obtain greater average returns than the current ones. One option could be to design a moderate **E45% fund with convergence 5 years before retirement.** There are other options, and the final choice will depend on the target returns and tolerable volatilities.

We will now calculate the replacement rates obtained for the different scenarios of rates of return and contribution densities, as the main objective of obtaining the greatest possible returns is to obtain the best replacement rates possible.

b.4. Replacement rates[16]

The replacement rate allows an assessment of compliance with the objective of the sufficiency of a pension system, to the extent that it indicates the capacity of members of the system to maintain their standard of living during active life in old age. This rate depends on variables within the pension system but also, and to a large extent, on variables outside it, such as those related to the characteristics of the labor market and the cycle of economic activity, among others. We will now simulate the replacement rates obtained for the Colombian case, with different scenarios of rates of return and contribution densities. Table 9 contains the results of these simulations. The most striking result is the impact contribution densities have on the replacement rates. A person who begins to contribute at 18 years of age and does so until 62, who presents a contribution density of 96% and a return of 7%, manages to replace 100% of his income at retirement. However, a person in the same conditions but with a contribution density of 3.7% manages a replacement rate of barely 4% of his last wage.

Table 9

Replacement rate for different contribution densities and returns (percentage of last wage)

Density of contributions	5	6	7	8	10	11	12	14
A	59%	76%	99%					
B	43%	56%	73%	97%				
C	17%	22%	28%	37%	66%	89%	120%	
D	2%	3%	4%	5%	9%	13%	16%	30%

Assumptions: 11.5% contribution rate, 2% real salary increase rate and density of contributions at:
A: 96%; B: 70.8%; C: 27.3% and D: 3.7%
Source: BBVA Research

16: The replacement rate is measured in this document as the proportion of the last wage received in active life.

In this sense, the quality of coverage (considered as the contribution density) is of prime importance in terms of the results of a pension system. Even with real rates of return of an annual average of 14%, the replacement rate for low densities would be very low, at 30% in the case of a density of 3.7%. In fact, high coverage in aggregate terms is no guarantee of good quality coverage. This is one of the points that led to a reform of the Chilean pension system, as, although the coverage rate in aggregate terms before the reform was adequate, the same was not true of its quality. At the time, it was estimated that close to 50% of pension fund members would not even achieve a minimum pension.

We should also point to another determining factor for the replacement rate, in this case the return. It can be presumed that determining a target rate of return is of prime importance when designing investment rules. However, although the target is set according to returns to be obtained, it is done based on certain basic assumptions that may not be realized.

4. Conclusions

The core objective of the study has been to carry out a theoretical exercise focused on the new investment rules for multi-funds by simulating aspects related to the composition of the portfolio in terms of its proportion of equity and fixed-income. It is important to point out that the results obtained in the different scenarios should be understood as part of the theoretical exercises based on certain assumptions and limited by the characteristics of the information available. Thus the patterns shown by our projections should be considered as reference and not as precise paths that the Colombian pension system will take under a multi-fund scheme. It should also be taken into account that at the completion date of the study, some central points of the law still had to be defined.

First, we can see that the investment horizon should be long-term to put any short-term performance into a more appropriate dimension. The Colombian experience shows that although there were losses in the first 10 months of 2008, these losses were more than offset during 2009. In the long term the returns in the industry have been very satisfactory.

Second, the simulations show that the investment rules have a significant influence on potential returns and risk management. The introduction of the multi-fund system in Colombia, with more diversified portfolios including equity, would be very positive in terms of returns, with volatility under control. The simulations show that members can obtain higher returns through the implementation of the multi-funds than with the single fund system, depending on the proportion of equity included in the funds. In carrying out the simulation in the Mexican case, whose riskiest fund contains 30% of equity compared with the case of the single fund with 30% of equity, the results obtained show lower returns in the Mexican case. This simulation shows that the life-cycle system limits returns more still. So with the aim of increasing returns by introducing the multi-funds, the riskiest fund in Colombia should have a higher proportion of equity than the current single fund. Similarly, it is to be expected that the moderate fund would have a higher equity component than the current single fund, given that this fund constitutes the default fund and that the convergence rule reduces returns. In fact, if the moderate fund is maintained with the proportion of equity of the current fund, this would presumably imply that the mere implementation of the multi-fund system would reduce the amount of equity in the system, as older people would enter directly with their assets into the conservative fund, whose proportion of equity is lower than the current fund and thus the average return would be reduced.

When the alternative convergence 10 years before retirement age is considered for the riskiest fund (80% equity), it was established that this scenario would be worse in terms of returns and Sharpe ratio than the above scenario with a transition 5 years before retirement, but better in terms of lower volatility. However, this convergence rule is also applied to the moderate fund, and should therefore be assessed for this fund as well. As an exercise, the convergence 10 years before retirement age was simulated for a default fund with a composition of **45% equity.** It was established that this option is inferior to the current scenario of **40% equity** in terms of average return, volatility and the Sharpe ratio. Thus the convergence rule simulated in the study for transition 10 years before retirement age for the conservative fund was rejected as an option. The option of **45% equity with 5-year convergence** is better than the current situation in terms of returns (better by 2.5%), with a higher volatility of 5.7%. The efficiency of the different possibilities should be examined in terms of the return/volatility ratio, and always on the basis of a target return with limited volatility.

On simulating the **E80%-E45% scenario with 5-year convergence**, the real average return obtained for the period of 39 years (women) would be **12.23%**, with an annual volatility of **3.01%** [17]). In the case of men, in scenario 1 **-E80%-E45%, no 5-year convergence rule-** after 44 years the return obtained would be **12.09%**, with a volatility of **2.76%** for the period. In this scenario, both in the case of women and men, a higher ratio is obtained between the annual average accumulated return and its volatility than would be obtained in the case of **E80% with convergence.** It may be thought that, although the return in the scenario **E80%-E45%, no convergence rule**, is lower than the scenario **E80% with convergence,** it is still high and with a higher average return per risk-unit ratio (4.1 compared with 3.9 after 39 years and 4.4 compared with 4.2 after 44 years). In addition, these returns are greater than those in the current **E40%** scenario (in this scenario the returns are around 10%) and better than those of the **E45%** scenario, which were simulated as default funds.

As mentioned above, in Colombia, the moderate fund is the default fund, which is why its design is so important. If the sole fund is maintained with its current composition (**E40% without convergence**) the real accumulated average annual return after 39 years would be 10% (not counting the reduction due to the convergence rule). The portfolio of **E45% without convergence** would give a real accumulated

17: In this scenario, 88% of the real returns obtained on 250 different paths are between 8.3% and 18%, and 17% of them are 10.5%.

average annual return of 10.73% after 39 years. In the case of the **E45% portfolio with convergence** the accumulated return (10.23% real annual average) would be in an intermediate position between **E40%** without convergence and **E45%** without convergence. In this scenario the return would be intermediate, as would the volatility.

To sum up, the moderate portfolio should probably have a greater proportion of equity in the Colombian case than the current single fund to obtain greater average returns than those at the present. One option could be the design of a moderate **E45% fund with 5-year convergence**. There are other options, and the final choice will depend on the target returns and tolerable volatilities. It should be pointed out, however, that the return obtained in the default portfolio would in this case be 15.5% (women) and 14.2% (men) lower than in the **E80%-E45%, no convergence rule**.

When calculating the replacement rates obtained for different scenarios of returns and contribution densities, the conclusion is that although the returns are of basic importance in determining the rates, the quality of coverage (seen as the contribution density) is key in terms of the results of a pension system. Even with real rates of return of an annual average of 14%, the replacement rate for low densities would be very low, at 30% in the case of a density of 3.7%. It is thus important to take into account that the performance of pension systems depends to a great extent on variables that lie outside of it. Variables such as the quality of coverage (contribution density) are crucially affected by the characteristics of the labor market and thus improvements in them depend largely on decisions outside the pension system.

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